Solutions to JEE Advanced Booster Test – 2|JEE 2024

PHYSICS

SECTION-1

Single Choice Type

1.(A) $\overrightarrow{v_B} = 8\hat{j} + 2t\hat{k}$

$\overrightarrow{v_c} = \overrightarrow{v_0} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$

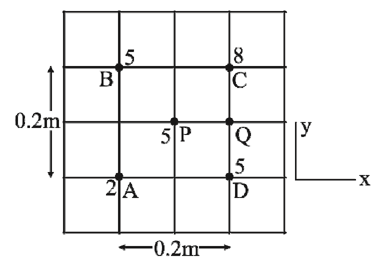
$\overrightarrow{r_B} = 8t\hat{j} + t^2\hat{k}$

$\overrightarrow{r_c} = v_x t\hat{i} + v_y t\hat{j} + v_z t\hat{k}$

At 4 sec, $\overrightarrow{r_B} = \overrightarrow{r_C} \Rightarrow v_x = 0, v_y = 8\,m/s$ and $v_z = 4m/s$ $\quad\therefore\quad$ $\overrightarrow{v_0} = (8\hat{j} + 4\hat{k})\,m/s$

2.(B) $V_A = 2$ volt.

$V_P = V_B = V_D = 5$ volt.



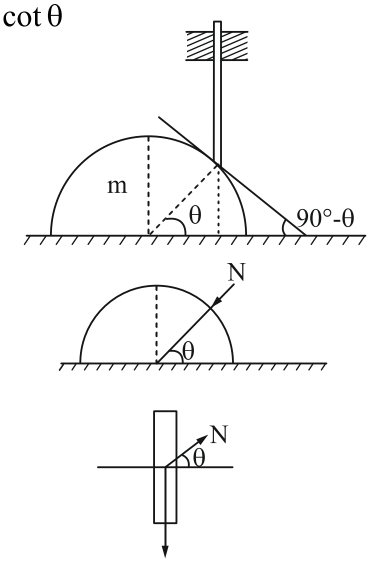
Since field is uniform

So $\quad E = \dfrac{\Delta V}{\Delta r}$; $\vec{E}_x = \dfrac{2-5}{0.2} = -15\hat{i}$ and $\vec{E}_y = \dfrac{2-5}{0.2} = -15\hat{j}$

Resultant field $\vec{E} = -15(\hat{i} + \hat{j})$

$E = 15\sqrt{2}$ and direction is along $\overrightarrow{PA}$

3.(D) If a be acceleration of the hemisphere, then acceleration of the rod is $a\cot\theta$



FBD of hemisphere

$N\cos\theta = Ma$

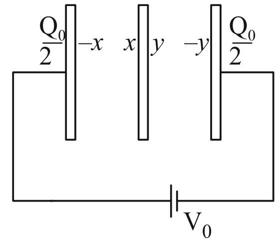
$mg - N\sin\theta = ma\cot\theta$

$mg - Ma\tan\theta = ma\cot\theta$

$mg = (M\tan + m\cot\theta)a$

$a = \dfrac{mg}{M\tan\theta + m\cot\theta}$

4.(B) **1.** $x+y=Q_0$



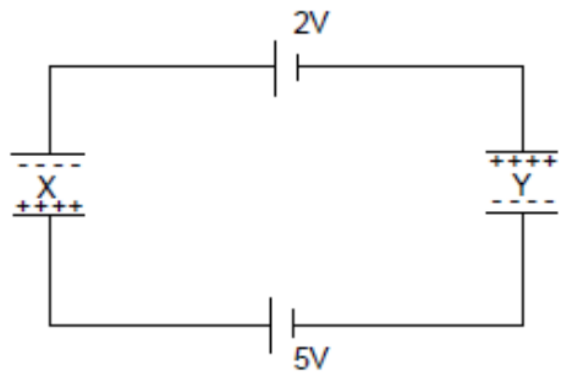
2. $\dfrac{-x(2d)}{A\in_0}+\dfrac{yd}{A\in_0}=V_0 \Rightarrow y-2x=\dfrac{A\in_0 V_0}{d}$

$$3x=Q_0-\frac{A\in_0 V_0}{d} \Rightarrow x=\frac{Q_0}{3}-\frac{A\in_0 V_0}{3d}$$

Charge on plate $I=\dfrac{Q_0}{2}-x=\dfrac{Q_0}{2}-\dfrac{Q_0}{3}+\dfrac{A\in_0 V_0}{3d}$

$$\frac{Q_0}{6}+\frac{A\in_0 V_0}{3d}$$

5.(B) Charge on each capacitor is $q=\dfrac{3C}{2}$



Electric field between plates $=\dfrac{q}{A\varepsilon_0}$

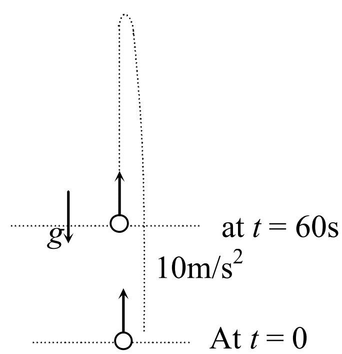
$$V_x+\frac{Ed}{2}-5+\frac{Ed}{2}=V_y$$

$$V_y-V_x=Ed-5$$

$$=\frac{q}{\varepsilon_0 A}d-5=\frac{q}{C}-5=\frac{3C/2}{C}-5=\frac{3}{2}-5=-3.5V$$

Multiple Correct Type

6.(BD) Total height $=\dfrac{1}{2}\times 10\times(60)^2+\dfrac{(10\times 60)^2}{2\times 10}$ = 36000 m = 36 km



Total time $=60+60+\sqrt{\dfrac{2\times 36000}{10}}$

$$=60s+60s+60\sqrt{2}s$$

$$=\left(120+60\sqrt{2}\right)\text{s}$$

7.(AD) For volt meter

$R+G=nG \quad \Rightarrow \quad 27=(n-1)G$

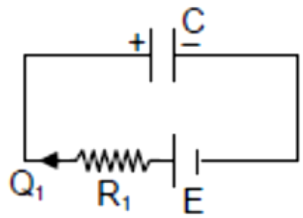
For ammeter

$G=(n-1)S \quad \Rightarrow \quad G=3(n-1)$

$$(n-1)^2=9$$

$$n=4$$

8.(AB) $Q_1 = CE$; $W_b = CE^2$



$U_1 = \frac{1}{2}CE^2$; $H_1 = \Delta H = \frac{1}{2}CE^2$

Charge flow through $E_2 = Q = 2CE$

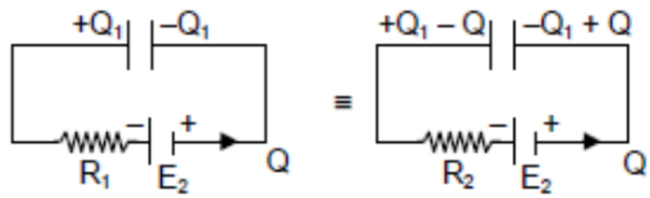
$W_b = 2CE^2$

$U_2 = \frac{1}{2}CE^2$

$\Delta U = 0$

$H_2 = 2CE^2$

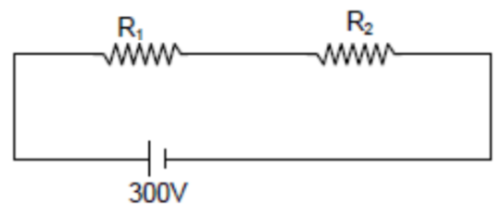
$\frac{H_1}{H_2} = \frac{1}{4}$



9.(ABCD) $\frac{(200)^2}{R_1} \times t_1 = 2 \times 6000 \times 20 \Rightarrow \frac{(200)^2}{R_1} = 400\,\text{watt} \Rightarrow R_1 = 100\Omega$

$\frac{(400)^2}{R_2} \times t_2 = 4 \times 12000 \times 20 \Rightarrow \frac{(400)^2}{R_2} = 800\ \text{watt}$

$\Rightarrow \quad R_2 = 200\,\Omega$

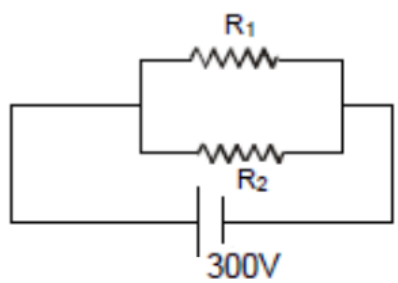


In series $\frac{(300)^2}{R_1 + R_2} \times t = 2 \times 6000 \times 30 \Rightarrow \quad t = 20\,\text{min}$

In Parallel

$\frac{(300)^2}{R_1R_2 / R_1 + R_2} \times t = 4 \times 12000 \times 27$

$t = 16\,\text{min}$



10.(B) When S is open:

$R_{AB} = \int_0^{\ell} \frac{3R_0 x dx}{\ell^2} = \frac{3R_0}{2}$; $I_0 = \frac{E_0}{R_0 + \frac{3R_0}{2}} = \frac{2E_0}{5R_0}$

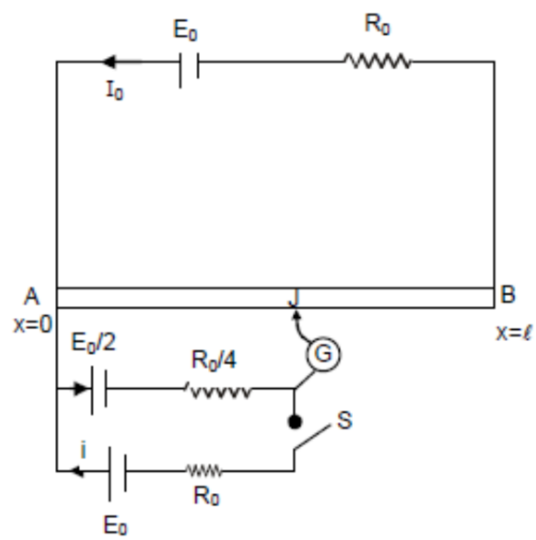
$\frac{E_0}{2} = I_0 = R_{AJ} = \frac{2E_0}{5R_0} \times \frac{3R_0}{\ell^2} \int_0^x x dx$; $x = \ell\sqrt{\frac{5}{6}}$

When s is closed

Current in secondary circuit, $i = \frac{\frac{E_0}{2}}{R_0 + R_0/4} = \frac{2E_0}{5R_0}$

Terminal voltage of battery $= \left[\varepsilon_0 - \frac{2\varepsilon_0}{5}\right] = \frac{3\varepsilon_0}{5}$

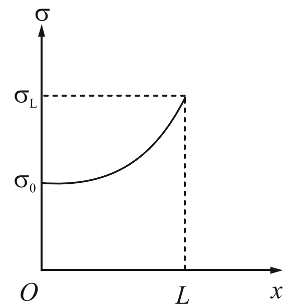
$\frac{3\varepsilon_0}{5} = I_0 \times R_{AJ} = \frac{2\varepsilon_0}{5R_0} \times \frac{3R_0}{\ell^2} \times \frac{x^2}{2} \Rightarrow x = \ell$



PARAGRAPH FOR Q-11 & 12

11.(B) 12.(A)

Consider a plate width of dx at a height x



Capacitance of this segment is $\Delta C = \frac{\epsilon_0 \Delta A}{d} = \frac{\epsilon_0 e^{\beta x} L \Delta x}{d}$

Potential difference across this small capacitor is V.

$\therefore \quad \Delta q = V \Delta C$

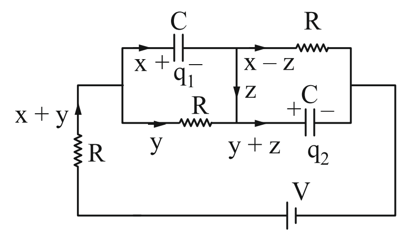
$\therefore \quad \sigma = \frac{\Delta q}{\Delta A} = \frac{V \Delta C}{L \Delta x} = \frac{V \epsilon_0 e^{\beta x}}{d}$

Graph is as shown $\sigma_0 = \frac{V \epsilon_0}{d}$ and $\sigma_L = \frac{V \epsilon_0}{d} e^{\beta L} = \sigma_0 e^{\beta L}$; Electric field remains constant at $E = \frac{V}{d}$

SECTION-2

1.(1.39) At time t let the charge on capacitors and current in different branches be as shown

$R(x+y) + Ry + R(x-z) = V$



$\Rightarrow \quad 2x + 2y - z = \frac{V}{R} \quad \ldots \text{(i)}$

$\frac{q_1}{C} = Ry \quad \ldots \text{(ii)}$

$\frac{q_2}{C} = R(x-z) \quad \ldots \text{(iii)}$

$\frac{dq_1}{dt} = x \quad \ldots \text{(iv)}$

$\frac{dq_2}{dt} = y + z \quad \ldots \text{(v)}$

From (ii) $\quad \frac{dq_1}{dt} = RC\frac{dy}{dt} \Rightarrow x = RC\frac{dy}{dt} \quad \ldots \text{(vi)}$

From (iii) $\quad \frac{dq_2}{dt} = RC\left(\frac{dx}{dt} - \frac{dz}{dt}\right) \Rightarrow y + z = RC\left(\frac{dx}{dt} - \frac{dz}{dt}\right) \quad \ldots \text{(vii)}$

(vi) + (vii) $\quad \frac{x+y+z}{RC} = \frac{dx}{dt} + \frac{dy}{dt} - \frac{dz}{dt} \quad \ldots \text{(viii)}$

From equation (i) $\quad x + y = \frac{V}{2R} + \frac{z}{2}$ also $\frac{dx}{dt} + \frac{dy}{dt} = \frac{1}{2}\frac{dz}{dt}$

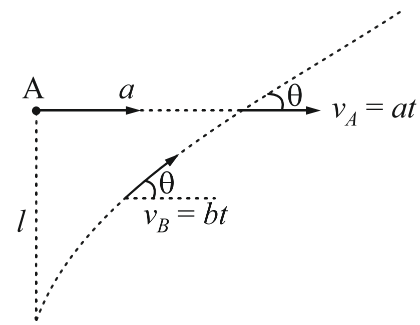
Unsing in (viii) $\quad \frac{V}{2R} + \frac{z}{2} + z = \left[\frac{1}{2}\frac{dz}{dt} - \frac{dz}{dt}\right]RC \; ; \; \frac{V}{R} + 3z = -RC\frac{dz}{dt}$

$$\int_{z=\frac{V}{R}}^{z} \frac{dz}{\frac{V}{R} + 3z} = -\frac{1}{RC}\int_{x=0}^{x} dt \qquad \left[\ell n\left(\frac{V}{R} + 3z\right)\right]_{\frac{V}{R}}^{z} = -\frac{3t}{RC}$$

$$\ell n\left(\frac{V}{R} + 3z\right) - \ell n\left(\frac{4V}{R}\right) = -\frac{3t}{RC} \; ; \; \ell n\left(\frac{1}{4} + \frac{3Rz}{4V}\right) = -\frac{3t}{RC} \; ; \; z = \frac{4V}{3R}\left[e^{\frac{-3t}{RC}} - \frac{1}{4}\right]$$

$$z = 0 \text{ when } e^{-\frac{3t}{RC}} = \frac{1}{4} \; ; \; \frac{3t}{RC} = \ell n 4 \; ; \; t = \frac{2}{3}RC\,\ell n(2)$$

2.(9) Let after time t, the velocity B is directed at angle θ with the horizontal



$$\frac{ds}{dt}=bt-at\cos\theta \Rightarrow -\int_l^0 ds=b\int_0^t t\,dt-a\int_0^t t\cos\theta dt$$

$$\frac{1}{2}at^2=b\int_0^t t\cos\theta\, dt$$

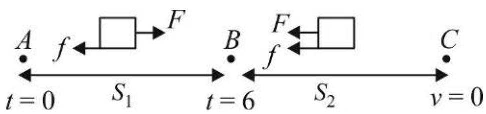
$$\therefore \qquad l=\frac{bt^2}{2}-\frac{a^2t^2}{2b};\quad t=\sqrt{\frac{2bl}{b^2-a^2}}$$

$$S=\frac{1}{2}bt^2=\frac{1}{2}b\frac{2bl}{b^2-a^2}=\frac{b^2l}{b^2-a^2}=9m$$

3.(2) $x=5t$ and $y=15t-\frac{1}{2}\times 10t^2$

At every point on plane, $x=y$ $\therefore$ $5t=15t^2 \Rightarrow t=2s$

4.(600)



From $A(t=0)$ to $B(t=6)$

$$a=\frac{F-f}{m}=10^{-5}=5\text{ m/s}^2\ ;\quad s_1=\frac{1}{2}at^2=\frac{1}{2}(5)(36)=90\text{ m}$$

Velocity at $B=at=5(6)=30$ m/s

After B, the direction of F is reversed.

As velocity is in forward direction, friction acts backwards till block comes to instantaneous rest at C.

From B to C

$$a=-\frac{(F+f)}{m}=-(10+5)=-15\text{ m/s}^2$$

$$u=30\text{ m/s}\ ;\ v^2=u^2+2as_2\ ;\ 0=(30)^2+2(-15)s_2 \quad\Rightarrow\quad s_2=30\,\text{m}$$

While returning from C to A, F acts towards left but friction acts towards right (as velocity is towards left)

$$s=-(s_1+s_2)=-120\text{ m}\ ;\quad s=-\frac{(F-f)}{m}=-5\text{m/s}^2$$

$$v^2=u^2+2as=0+2(-120)(-5)\ ;\ v^2=1200 \quad\Rightarrow\quad v=20\sqrt{3}$$

5.(21.33) $a=2-0.5t$; $\frac{dv}{dt}=2-0.5t$; $\int_0^v dv=\int_0^t(2-0.5t)\,dt$

$$v=2t-0.25\,t^2\ ;\ v=0\text{ at } t=0 \text{ and } t=8\ ;\ \frac{dx}{dt}=2t-0.25\,t^2$$

$$\int_0^x dx=\int_0^t(2t-0.25\,t^2)\,dt\ ;\ d=\left[t^2-\frac{t^3}{12}\right]_0^8=64-\frac{512}{12}=21.33\,m$$

6.(–48) $\vec{E}=4x\hat{i}-(y^2+1)\hat{j}$; For ABCD: $\overrightarrow{dA}=(dA)\hat{k} \Rightarrow \vec{E}\cdot\overrightarrow{dA}=0$ $\therefore$ $\phi_1=0$

For BCGF : $\overrightarrow{dA}=(dA)\hat{i}$ $\therefore$ $\phi_2=\int[4x\hat{i}-(y^2+1)\hat{j}].(dA\hat{i})$

$$=\int 4x\,dA=\int 4(3)dA=12A\ ;\quad =12(4)=48\ ;\quad \phi_1=\phi_2=-48$$

CHEMISTRY

SECTION-1

Single Choice Type

1.(A) Lyophilic sols do not show significant Tyndall effect because sol particles are very small and highly solvated. There is almost no difference is the refractive indices of dispersion medium and dispersed phase.

2.(D) According to Arrhenius rate constant increases with temperature.

3.(C) At anode $2Hg + 2Cl^- \rightleftharpoons Hg_2Cl_2 + 2e^-$

At cathode $\dfrac{Q + 2H^+ + 2e^- \rightleftharpoons QH_2}{2Hg + Q + 2H^+ + 2Cl^- \rightleftharpoons QH_2 + Hg_2Cl_2}$

$E_{Cell} = 0.183\ V \qquad E^{\circ}_{Cell} = 0.6990 - 0.42 = 0.2790$

$$E_{Cell} = E^{\circ}_{Cell} \frac{-0.06}{n} \log \frac{[\cancel{QH_2}]}{[Cl^-]^2 [H^+]^2 [\cancel{Q}]}$$

$$0.183 = 0.279 - \frac{0.06}{2} \log \frac{1}{[H^+]^4}$$

$pH = 0.8$

4.(B) Given values of ionisation enthalpies and electron gain enthalpies suggest that element X belongs to 13th group, element Z belongs to 2nd group $\left(\because (+)\text{ve value of } \Delta H^{\circ}_{eg}\right)$ and element Y is probably a member of 16th or 17th group.

5.(C) Rate = k' $[CH_3COOCH_3]$ $[H_2O]$

For pseudo first order reaction, the reaction should be first order with respect to ester when $[H_2O]$ is constant.

The rate constant k for pseudo first order reaction is $k = \dfrac{2.303}{t} \log \dfrac{C_0}{C}$, where $k = k'[H_2O]$

From the above data we note:

t/min	C/mol L^{-1}	k/mol L^{-1}
0	0.8500	–
30	0.8004	2.004×10^{-3}
60	0.7538	2.002×10^{-3}
90	0.7096	2.005×10^{-3}

It can be seen that $k[H_2O]$ is constant and equal to $2.004 \times 10^{3}\ min^{-1}$ and hence, it is pseudo first order reaction. We can now determine k from

$k\,[H_2O] = 2.004 \times 10^{-3}\ min^{-1}$

$k[55\ mol\,L^{-1}] = 2.004 \times 10^{-3}\ min^{-1}$

$k = 3.64 \times 10^{-5} mol^{-1}\ L\ min^{-1}$

Multiple Correct Type

6.(ABC) Statement A, B and C are correct. Proteins can be dissolved in water thus all protein sol are not Lyophobic.

7.(ABD) In single electron atomic species, Z_{eff} for any position of electron is equal to Z. Balmer series of Be^{3+} does not belongs to the visible region.

8.(ABD) Orders given in option A, B and D are correct. Atomic size of Al is slightly bigger than that of Ga.

9.(AC) $\log K_{eq} = \dfrac{nFE^\circ}{2.303\,RT}$

$$E^\circ_{Cell} = E^\circ_{Ag^+|Ag} - E^\circ_{Fe^{3+}|Fe^{2+}} = 0.0296\text{ V}$$

$$\log K_{eq} = \frac{1\times 0.0296}{\frac{2.303RT}{F}} = \frac{1}{2}$$

$$K_{eq} = 3.16$$

$$Q = \frac{\left[Fe^{3+}\right]}{\left[Fe^{2+}\right]\left[Ag^+\right]} = 1 \qquad \because \qquad Q < Keq$$, so reaction will move in forward direction.

$$Fe^{2+} + Ag^+ \rightleftharpoons Ag + Fe^{3+}$$

$$\begin{array}{lcccc} & Fe^{2+} & +Ag^+ \rightleftharpoons & Ag & +Fe^{3+} \\ t=0 & 1 & 1 & - & - \\ t=eq & 1-x & 1-x & x & x \end{array}$$

$$3.16 = \frac{x}{(1-x)^2}$$

$$x = 0.574\text{ M} = \left[Fe^{3+}\right]$$

$$\left[Ag^+\right] = \left[Fe^{2+}\right] = 0.426\text{ M}$$

10.(AB) In $Ti_{0.96}O$

$$\frac{Ti^{3+}}{Ti^{2+}} = \frac{2-2(0.96)}{3(0.96)-2} = \frac{0.08}{2.88-2} = \frac{1}{11}$$

$$\Rightarrow \text{ moles of } Ti^{2+} = \frac{11}{12}\text{ moles}$$

$$\Rightarrow \text{ moles of } Ti^{3+} = \frac{1}{12}\text{ moles}$$

Gram equivalent of reducing agent $= \dfrac{11}{12}(2) + \dfrac{1}{2}(1) = \dfrac{23}{12} \Rightarrow y\times 5 = \dfrac{23}{12}$ (or) $y = \dfrac{23}{60} = 0.383$ moles

for TiO ; $y \times 5 = 2\times 1$; $y = 0.40$ moles

PARAGRAPH FOR Q-11 & 12

11.(C) $E = 0.4 - 2\times10^{-4}\times300$

$E = 34\times10^{-2}$

$E = 0.34\ V$

12.(C) $\Delta S = 1\times96500\times\left(-2\times10^{-4}\right)JK^{-1}\ mol^{-1}$

$\Delta S = -19.3\ JK^{-1}\ mol^{-1}$

SECTION-2

1.(28) $Cu + 2Ag^{+} \rightleftharpoons Cu^{+2} + 2\ Ag$

$$Q_c = \frac{[Cu^{+2}]}{[Ag^{+1}]^2}$$

For $[Cu^{+2}] = 0.1\ M, [Ag^{+}] = 0.01\ M$

$$E_{cell} = E^o_{cell} = \frac{0.059}{2}\log\frac{[Cu^{+2}]}{[Ag^{+}]^2}$$

$$0.3095 = E^o_{cell} - \frac{0.059}{2}\log_{10}\frac{10^{-1}}{10^{-4}} \quad ; \quad 0.3095 = E^o_{cell} - 1.5\,[0.059]$$

$$\therefore\ E^o_{cell} = 0.3095 + 1.5[0.059]$$

For $[Cu^{+2}] = 0.01,\ [Ag^{+}] = 10^{-3}\ M$

$$E_{cell} = 0.3095 + 1.5[0.059] - \frac{0.059}{2}\log\frac{10^{-2}}{10^{-6}} \quad = 0.3059 + 1.5[0.059] - 2[0.059]$$

$$= 0.3095 - \frac{0.059}{2} = \left[30.95 - \frac{5.9}{2}\right]\times10^{-2} \quad = 28\times10^{-2}$$

2.(5) Factual As_2S_3 is negatively charged colloid.

3.(6) **(i)** Moles of the gas :

$$^{n}(gas) = \frac{V(gas)\ (L)}{V_m} = \frac{2.46\ (L)}{24.6\ (L/mole)} = 0.1\ mole$$

(ii) $$moles = \frac{W(sample)}{Molar\ mass}$$

$$Molar\ mass = \frac{W(sample)}{Moles} = \frac{24.6}{0.1} = 246\ g/mole$$

Molecular mass of the $XeF_n = 246$

(iii) Mass of 1 molecule = sum of masses of the atoms

$246 = m_{xe} + n[m_F];\ \ 246 = 132 + n[19]$

$n = 6 \quad (\therefore XeF_6)$

4.(3) At radial node

$R_{3p} = 0$

$(6-\sigma) = 0$

$\Rightarrow \quad \sigma = 6$

(or) $\quad \dfrac{r}{a_0} = 3 \qquad (\text{for } He^+ \text{ ion})$

5.(7) $\dfrac{kt}{ko}$ is temperature coefficient, when $T = 10°C$

so $\quad \ell n\left(\dfrac{kt}{ko}\right) = \left(\dfrac{T}{283}\right)\ell n 7$

$$\ell n\left(\frac{kt}{ko}\right) = \frac{283}{283} \times \ell n 7$$

$$\ell n\left(\frac{kt}{ko}\right) = \ell n 7$$

$$\Rightarrow \quad \frac{kt}{ko} = 7$$

6.(5) Reduction potential is – 0.3 V

$$2H^+ + 2e^- \longrightarrow H_{2(g)}$$

so, $\quad E_{H^+/H_2} = E°_{H^+/H_2} - \dfrac{0.06}{n} \log \dfrac{1}{\left[H^+\right]^2}$

$\Rightarrow \quad -0.3 = 0 - \dfrac{0.06}{2}\left(-2\log\left[H^+\right]\right)$

$\Rightarrow \quad -0.3 = 0 - \dfrac{0.06}{2}(pH) \times 2$

so, $\quad pH = \dfrac{0.3}{0.06} = 5$

MATHEMATICS

SECTION-1

Single Choice Type

1.(A) $y = 2\ln(1+\cos x)$

$$y_1 = \frac{-2\sin x}{1+\cos x};\; y_2 = -2\left[\frac{(1+\cos x)\cos x - \sin x(-\sin x)}{(1+\cos x)^2}\right] = -2\left[\frac{\cos x+1}{(1+\cos x)^2}\right] = \frac{-2}{(1+\cos x)}$$

$$\therefore\; 2e^{-y/2} = 2\cdot e^{-\frac{\ln(1+\cos x)^2}{2}} = \frac{2}{(1+\cos x)} \qquad \therefore\; y_2 + \frac{2}{e^{y/2}} = 0$$

2.(B) Let the slope of L be m. L makes an angle of 60º with $\sqrt{3}x + y = 1$, hence

$$\tan 60^\circ = \left|\frac{m+\sqrt{3}}{1-\sqrt{3}m}\right| = \sqrt{3} \Rightarrow \frac{m+\sqrt{3}}{1-\sqrt{3}m} = \sqrt{3} \text{ or } -\sqrt{3} \Rightarrow \quad \text{Either } m+\sqrt{3} = \sqrt{3}-3m \text{ or } m+\sqrt{3} = 3m-\sqrt{3}$$

Now, $m = 0$ (rejected as L is not parallel to x-axis)

$\therefore$ Equation of L is $y+2 = \sqrt{3}(x-3) \Rightarrow y - \sqrt{3}x + 2 + 3\sqrt{3} = 0$

3.(D) **(A)** $f : R-\{0\} \to R,$

$f'(x) = 1 - \dfrac{1}{x^2}$ can be positive or negative

$\Rightarrow f(x)$ is not one-one

Also, range = $[-\infty,-2] \cup [2,\infty) \neq$ co-domain

$\Rightarrow f(x)$ is not on to

(B) $f : R \to R,\; f(x) = x^3 + x^2 + 3x + \sin x$

$\Rightarrow f'(x) = 3x^2 + 2x + 3 + \cos x$

$$= \underbrace{\left(3x^2+2x+1\right)}_{a>0,\, D<0} + \underbrace{(2+\cos x)}_{\text{always positive}} > 0\; f \text{ is one-one and on to}$$

(C) $f : R \to R,\; f(x) = \left[x+\frac{1}{2}\right] + \left[x-\frac{1}{2}\right] + 2[-x]$

$$= \left(x+\frac{1}{2}\right) - \left\{x+\frac{1}{2}\right\} + \left(x-\frac{1}{2}\right) - \left\{x-\frac{1}{2}\right\} + 2\left(-x-\{-x\}\right)$$

$$= -\left\{x+\frac{1}{2}\right\} - \left\{x-\frac{1}{2}\right\} - 2\{-x\}$$

$\Rightarrow$ Periodic with T.P. 1

(D) $f : R \to R,\; f(x) = e^{\sin\{x\}} + \sin\left(\frac{\pi}{2}[x]\right)$, $e^{\sin\{x\}}$ is periodic with fundamental period 1 and $\sin\left(\frac{\pi}{2}[x]\right)$ is periodic with fundamental period 4, therefore $f(x)$ is periodic with fundamental period 4

4.(A) Let $S=\sum_{r=1}^{n}\tan^{-1}\left(\frac{4}{4r^2+3}\right)$

$$=\sum_{r=1}^{n}\tan^{-1}\left(\frac{1}{1+\left(r+\frac{1}{2}\right)\left(r-\frac{1}{2}\right)}\right)$$

$$=\sum_{r=1}^{n}\tan^{-1}\left(\frac{\left(r+\frac{1}{2}\right)-\left(r-\frac{1}{2}\right)}{1+\left(r+\frac{1}{2}\right)\left(r-\frac{1}{2}\right)}\right)=\sum_{r=1}^{n}\left[\tan^{-1}\left(r+\frac{1}{2}\right)-\tan^{-1}\left(r-\frac{1}{2}\right)\right]$$

$$S=\tan^{-1}\left(n+\frac{1}{2}\right)-\tan^{-1}\left(\frac{1}{2}\right)$$

Now, $\lim_{n\to\infty}S=\frac{\pi}{2}-\tan^{-1}\frac{1}{2}=\cot^{-1}\frac{1}{2}=\tan^{-1}2$

$\therefore \tan\left(\tan^{-1}2\right)=2$

5.(B) $2[x]=\{x+5\}+x-1 \quad \Rightarrow 2[x]=\{x\}+x-1 \quad \Rightarrow x=-1,\frac{1}{2}$

Multiple Correct Type

6.(BCD)

(A) $f(x)=\frac{x}{e^x-1}+\frac{x}{2}+1 \quad \Rightarrow \quad f(-x)=\frac{-x}{e^{-x}-1}-\frac{x}{2}+1$

$$\Rightarrow f(-x)=\frac{-xe^x+x-x}{1-e^x}-\frac{x}{2}+1=\frac{x}{e^x-1}+x-\frac{x}{2}+1=f(x)$$

$$\Rightarrow f(-x)=f(x)\Rightarrow f \text{ is even}$$

(B) $f(x)=\begin{cases}0, & x\in Q\\ 1, & x\notin Q\end{cases}$ is periodic with No FTP

(C) If $f(x)$ is odd then $f(-x)=-f(x)$ which can be possible then $f(0)=0$

(D) $Sgn\left(\left[\frac{15}{1+x^2}\right]\right)=[1+\{2x\}]$

$$\Rightarrow Sgn\left(\left[\frac{15}{1+x^2}\right]\right)=1 \quad \Rightarrow \left[\frac{15}{1+x^2}\right]>0$$

$$\Rightarrow \frac{15}{1+x^2}\ge 1 \quad \Rightarrow x^2\le 14 \quad \Rightarrow -\sqrt{14}\le x\le\sqrt{14}$$

$\Rightarrow$ Integer value of x = –3, – 2, – 1, 0, 1, 2, 3

7.(ABCD)

(A) RHL

$$\lim_{x\to0^+} f(x)=\lim_{h\to0} f(h)\ =\lim_{h\to0}\sin\frac{\pi}{2h}$$ = A value oscillating between –1 and 1

i.e. RHL does not exist.

Similarly, LHL also does not exist $\therefore$ $f(x)$ is discontinuous as limit does not exist at $x=0$

(B) RHL

$$\lim_{x\to0^+} g(x)=\lim_{h\to0} g(h)=\lim_{h\to0} h\sin\left(\frac{\pi}{h}\right)$$

= 0 × (A value oscillating between –1 and 1) = 0

Similarly LHL = 0

But $\lim_{x\to0} g(x)\neq g(0)$ $\therefore$ $g(x)$ is discontinuous at $x=0$

(C) RHL

$$\lim_{x\to0^+} h(x)=\lim_{h\to0}\frac{|h|}{h}=\lim_{h\to0}\frac{h}{h}=1$$

LHL

$$\lim_{x\to0^-} h(x)=\lim_{h\to0}\frac{|-h|}{h}=\lim_{h\to0}\frac{h}{-h}=-1$$

As $LHL\neq RHL$ at $x=0$ $\therefore$ $h(x)$ discontinuous at $x=0$

(D) RHL

$$\lim_{x\to0^+} k(x)=\lim_{h\to0}\frac{1}{1+e^{\text{coth}}}$$ =0 (As $h\to0, e^{\cot h}\to\infty$)

LHL

$$\lim_{x\to0^-} k(x)=\lim_{h\to0}\frac{1}{1+e^{\cot(-h)}}\ \lim_{h\to\infty}\frac{1}{1+e^{-\cot h}}=1$$ (As $h\to0, e^{-\cot h}\to0$)

As $LHL\neq RHL$ at $x=0$ $\therefore$ $k(x)$ discontinuous at $x=0$

8.(ABCD) $f(x)=\cos^{-1}+\cos^{-1}\left(\frac{1}{2}.x+\sqrt{1-\left(\frac{1}{2}\right)^2}.\sqrt{1-x^2}\right)$

$$=\cos^{-1}x+\left|\cos^{-1}\frac{1}{2}-\cos^{-1}x\right|\ =\cos^{-1}x+\left|\frac{\pi}{3}-\cos^{-1}x\right|$$

$$=\begin{cases} 2\cos^{-1}x-\frac{\pi}{3}, & x\in\left[-1,\frac{1}{2}\right] \\ \frac{\pi}{3} & x\in\left[\frac{1}{2},1\right] \end{cases}$$

Now go through the options.

9.(ABC) $f: R \to \left[\frac{-\pi}{4}, \frac{\pi}{2}\right)$ $\quad f(x) = \tan^{-1}\left(\left(x^2 - \frac{1}{2}\right)^2 + \tan^{-1}\alpha - 2\right)$

for $f(x)$ to be surjective, $\tan^{-1}\alpha - 2 = -1 \;= \alpha = \tan 1$ Now check options.

10.(AC) $x^2 + y^2 + 8x - 10y - 40 = 0$

$(x+4)^2 + (y-5)^2 = 1$

$x = \cos\theta - 4,\ y = \sin\theta + 5$

$(x+2)^2 + (y-3)^2$

$= (\cos\theta - 2)^2 + (\sin\theta + 2)^2 = 9 + 4(\sin\theta - \cos\theta)$

$a = \max(9 + 4(\sin\theta - \cos\theta)) = 9 + 4\sqrt{2}$

$b = \min(9 + 4(\sin\theta - \cos\theta)) = 9 - 4\sqrt{2}$

$a + b = 18$

$a - b = 8\sqrt{2}$

$a \cdot b = 49$

PARAGRAPH FOR Q-11 & 12

Given $\sin^{-1}x : [-1, 1] \to \left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$ and $\cos^{-1}x : [-1, 1] \to [\pi, 2\pi]$

11.(C) Let $\sin^{-1}(-x)y$, $\quad y \in \left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$

$\Rightarrow -x = \sin y \;\Rightarrow x = -\sin y \;\Rightarrow x = \sin(2\pi - y)$

$\Rightarrow \sin^{-1}x = 2\pi - y \;\Rightarrow y = 2\pi - \sin^{-1}x \;\Rightarrow \sin^{-1}(-x) = 2\pi - \sin^{-1}x$

12.(C) $\cos^{-1}(\cos 10) - \sin^{-1}(\sin 10)$

$= \cos^{-1}(\cos(10 - 2\pi) - \sin^{-1}(-\sin(4\pi - 10)$

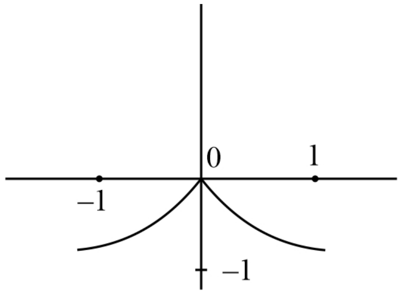
$= \cos^{-1}(\cos(10 - 2\pi) - 2\pi + \sin^{-1}(\sin(4\pi - 10))$

$= 10 - 2\pi - 2\pi + 4\pi - 10 = 0$

SECTION-2

1.(3) Graph of $y = f\left(-|x|\right)$ is :

$\Rightarrow$ Domain $= [-1, 1]$, Range = [-1, 0]



2.(3) $x^2+y^2-8x+2y+8=0$

$x^2+y^2-2x-6y+10-a^2=0$

$|r_1-r_2|<C_1C_2<r_1+r_2$

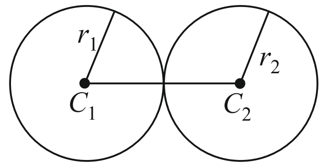
$C_1(4,-1),\ C_2(1,3)$

$r_1=\sqrt{16+1-8}=3$

$r_2=\sqrt{1+9-10+a^2}=|a|$

$C_1C_2=\sqrt{9+16}=5$

$\left|3-|a|\right|<5<3+|a|$ **;** $2<|a|<8$ **;** $a\in(-8,-2)\cup(2,8)$



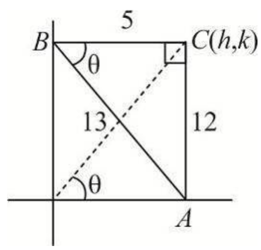
3.(0.50) Domain of $f\left(\left|[x]\right|\right)$, $-3\le\left|[x]\right|\le 2$

$\Rightarrow -2\le[x]\le 2\ \Rightarrow -2\le x<3$

$\Rightarrow x\in[-2,3)\ \ \Rightarrow a=-2, b=3$

4.(5) $\tan\theta=\dfrac{k}{h}=\dfrac{12}{5}$

$12x-5y=0\Rightarrow\quad k=5$



5.(9) $\displaystyle\lim_{x\to0}\frac{e^{\tan x}-e^x}{\tan x-x}=\lim_{x\to0}\frac{e^x\times e^{(\tan x-x)}-e^x}{\tan x-x}$

$\displaystyle\lim_{x\to0}\frac{e^x(e^{\tan x-x}-1)}{\tan x-x}=\lim_{x\to0,y\to0}\frac{e^x\times(e^y-1)}{y}$, where $y=\tan x-x$ and $\displaystyle\lim_{y\to0}\frac{e^y-1}{y}=1$

$=e^0\times1$ [as $x\to0,\ \tan x-x\to0$]

$=1\times1=1$

As $\tan 45^\circ=1$

$\therefore\ \ k=45$ sum of digits of k is 9

6.(7) Clearly x will be defined only for $a=0$ and then

$$x=\sin^{-1}(1)+\cos^{-1}(-1)-\tan^{-1}(1)$$

$$x=\frac{5\pi}{4}$$

Now $\cos^{-1}(\cos x)=\cos^{-1}\left(\cos\dfrac{5\pi}{4}\right)=\dfrac{3\pi}{4}$